PREFACE

Modulation spaces form a class of function spaces that measure smoothness in terms of phase-space (time-frequency) concentration instead of differences and derivatives. This class of function spaces was introduced by Hans G. Feichtinger in the early 1980s and has become an important area of research in recent years. One main reason for the modern flourishing of modulation spaces is that, simply put, they are the "right" function spaces for time-frequency analysis and phase-space analysis. They occur naturally throughout time-frequency analysis: in time-frequency expansions, in the formulation of fine uncertainty principles, in the construction of Weyl-Heisenberg frames (Gabor frames), and as symbol classes for pseudodifferential operators. The fact that the modulation spaces provide a good way to quantify time-frequency concentration and to describe time-varying systems is currently being exploited in such engineering applications as wireless communications.

A quick search in the *Mathematical Reviews* yields 49 articles (including a few book chapters) that deal with modulation spaces. The vitality of this research direction is evidenced by the fact that 36 of these articles appeared after 2000. Ironically, the two most influential groups of articles are not included in this list. Feichtinger's original preprint from 1983, "Modulation spaces on locally compact abelian groups," did not appear in print until recently. Yet the impressive development of time-frequency analysis would not be possible without Feichtinger's fundamental work, which develops the basic theory of modulation spaces not only on $\mathbb{R}^d$, but simultaneously on all locally compact abelian groups.

The second set of articles missing from this list are "Wiener type algebras of pseudodifferential operators" and "An algebra of pseudodifferential operators" from 1994 and 1995. In these articles, Johannes Sjöstrand introduced a particular modulation space as a non-standard symbol class for pseudodifferential operators. However, as he was unaware of the existing theory of modulation spaces, he named his function space differently, and so his work escapes a search for modulation spaces. Yet his papers have opened a new direction in time-frequency analysis. Both groups of papers, by Feichtinger and by Sjöstrand, continue to have a profound influence on research and are true landmarks in the field.

This special issue collects a number of excellent articles on modulation spaces that point towards new directions for time-frequency analysis and modulation spaces. They exhibit the vigor of this special mathematical topic 25 years after its invention. Feichtinger's algebra, a special case of a modulation space, was discovered in 1979 and his full theory of modulation spaces was presented in 1983; so, in an average sense, this special issue may be regarded as a celebration of the 25th birthday of modulation spaces.

The first article is comprehensive account of the history of modulation spaces and a detailed survey of the current literature, written by the master of modulation spaces himself, Hans G. Feichtinger. This is a first-person historical account of the invention and development of the modulation spaces, and we are honored to have Hans' contribution.

In this volume the modulation spaces occur both as a topic of intrinsic interest and as a tool for the study of other mathematical questions. The articles cover a wide range of topics in modulation spaces and time-frequency analysis. Guido Janssen gives a new characterization of the modulation space M^1 (currently called Feichtinger's algebra) by means of the Zak transform. Gitta Kutyniok uses modulation space techniques as a tool in her investigation of density conditions for non-uniform and weighted Gabor frames. Götz Pfander and David Walnut study the problem of system identification in communication theory and show that modulation spaces are the perfect context for solving a conjecture by Kailath. Holger Rauhut studies radial functions in modulation spaces and proves a number of surprising embedding results. Nenad Teofanov identifies Gelfand-Shilov spaces as projective limits of certain modulation spaces and gives an intriguing application to pseudodifferential operators.

We hope that the ideas developed in this small, but precious, volume will instigate further research and contribute to the health and growth of time-frequency analysis. We would like to thank the authors for their impressive contributions. A special thanks goes to Hans G. Feichtinger for his continued advice and expertise. Last, but not least, we would like to thank the Executive Editor, Abdul Jerri, for the opportunity to prepare this special issue for *Sampling Theory in Signal and Image Processing.*

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Karlheinz Gröchenig, Vienna
Christopher Heil, Atlanta, Georgia